Influence of the critical curvature on spiral initiation in an excitable medium

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Spiral waves are created in a Belousov-Zhabotinsky medium due to the presence of some inert obstacle placed into that medium. The eikonal equation, which is completely valid when the excitability of the system remains constant, is shown to be insufficient when media with different excitabilities are used, as predicted by Zykov's kinematical theory [Biophys. **25**, 906 (1980)], where two parameters, excitability and curvature, are varied.

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I. INTRODUCTION

A spiral wave in a two-dimensional medium represents a self-sustained activity. This kind of wave has been the object of exhaustive research for decades, both experimentally [1–3], analytically [4,5], and numerically [7–9] in reaction-diffusion systems [10].

In particular, spiral appearance has been widely studied in cardiac tissue [11,12] since it seems to be intimately related to some pathological states that disturb cardiac rhythm. Thus, insights may be gained by exploring conditions suitable for spiral wave initiation. In the literature, several mechanisms to generate spirals have been specially studied: *vulnerability* [12–15], which consists in delivering a stimulus immediately after the passage of a previous wave; or local inhibition, which consists in cutting a piece of the wave front by means of a temporal disturbance in the medium, creating a discontinuous wave front [16,17]. Another alternative method consists in disturbing the propagation of waves in a medium by means of the presence of permanent obstacles in that medium, e.g., a previously infarcted region in the heart [18,19]. Different kinds of interactions are possible when a wave train collides with an obstacle, waves can follow the edge of the obstacle and surround it or leave the edge, giving rise to a discontinuous wave front that may evolve in a rotating spiral.

The origin of the previously mentioned spiral formation must be investigated in terms of the existence of a critical curvature (maximal curvature permitted by a certain medium), above which a wave cannot propagate. Different efforts, both theoretical [20–22] and experimental [23,24], have been devoted to the understanding of this phenomenon. In particular, Foerster, Müller, and Hess [23] pursued experiments in a Belousov-Zhabotinsky (BZ) [25] medium, exploring an idea that had been previously suggested and numerically developed by Zykov and Morozova [20]. They initiated waves by immersing very thin glass capillarities spattered with a silver film in a liquid BZ medium, which acted as a chemical stimulus for a wave [26]. With different radius capillaries, they were able to find a critical radius below which wave expansion was not possible. Using the eikonal equation [22], they estimated the diffusion coefficient corresponding to their BZ reaction.

Although a lot of theoretical research concerning the understanding of spiral initiation in chemical systems has been done, in other fields, e.g., cardiology, spiral initiation is intimately related to the appearance of some pathologies, and it is there that this problem becomes worthy of attention. Recent clinical trials of drugs that affect cellular excitability in the heart were terminated after observing an increase in the rate of sudden cardiac death in the treated group relative to the untreated group [18]. All patients in this study had recently survived a myocardial infraction and may have had structural damage to heart tissue. Spiral appearance in such a medium is influenced both by the excitability of the medium and by the geometry of the damaged region.

Throughout this paper, we investigated the influence that excitability of a BZ medium has on the critical curvature, and we fitted the experimental results to the theoretical predictions obtained from a general reaction-diffusion system [21]. A brief discussion about the possibility of extending our results to other systems, e.g., the cardiac muscle, is also presented.

II. EXPERIMENTAL METHOD

Experiments were performed in a BZ medium, where the catalyst (ferroin $0.008M$) was immobilized in a silica gel [27]. The gel was 1-mm thick in a Petri dish of 49 mm in diameter. The Petri dish was filled with the following recipe: $\frac{1}{6}M$ $CH_2(COOH)_2$, $\frac{1}{6}M$ H_2SO_4, varying the $NaBrO_3$ concentration (from $0.04M$ to $0.35M$) to get different excitabilities [22] in the system. The depth of the liquid layer was always 6 mm to prevent any interference between the oxygen in the air and the BZ reaction. The experiments were performed at room temperature (25 ± 1 °C).

After preparing the gel, while it was still liquid, a piece of a chemically inert material (glass and Teflon slides

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were used with similar results) was placed in the middle of the Petri dish. In our experiments, objects of different geometries were used; although they always presented analogous properties, their curvature was smooth (almost zero) along most of the edge of the obstacle, except on its extremes, where a high curvature was imposed. These extremes presented a round shape, where the curvature of the wave was experimentally measured. Experiments were followed with a charge-coupled-device (CCD) camera and recorded on a video tape. Images were taken close to the tip and converted to a binary file. Near the obstacle edge, the position of those points belonging to the wave front were fitted to a circumference with an accuracy better than 98%, and the curvature of the wave front was estimated as the inverse of its radius. The particular value of the curvature, above which, for a certain excitability, the front leaves the edge of the obstacle, is called the *critical curvature* (K_c). Experimentally, obstacle radii between 75 and 300 μm were considered.

A circular wave was generated somewhere in the medium by touching the upper surface of the gel with a silver wire [26]. A part of this wave was inhibited by means of an iron wire, which gave rise to a discontinuous wave front that evolved into a pair of rotating spiral waves. This constituted our *test wave*. When the test wave reached the obstacle placed in the middle of the gel, different behaviors were observed depending on the curvature of the obstacle and the excitability of the medium. No refractoriness effects were observed, which was checked by decreasing the frequency of the spiral waves using a method similar to that described in [28] (a micro drop, about 15 μl, of 1M malonic acid was injected with a syringe into the core of the test wave).

The velocity V_0 of the test wave was measured in a part of the front far from its tip, where the curvature can be considered to be zero. On the other hand, the wave position close to the obstacle was followed and recorded along the experiment, which allowed us to calculate the velocity V. The particular value of this velocity, under which the front leaves the edge of the obstacle, is called *critical velocity* (V_c).

It would be possible to carry out the same experiment using a liquid system, but a silica gel was used in order to compare our results to those theoretically predicted in [21], where one of the variables (the so called *inhibitor)* had a diffusion coefficient equal to zero.

Capillarity effects close to the obstacle and to the walls of the Petri dish were considered negligible or, at least, they did not affect qualitatively the observed phenomena. The influence of the liquid and gel capillarity was studied by using different chemically inert obstacles (glass and Teflon slides, as was previously mentioned). The increase of the gel thickness close to the obstacle was always less than 2% of the gel thickness far from it. In experimental figures, this effect is seen as a shadow in the region near the obstacle.

III. RESULTS

When a wave collides with an obstacle, different responses can be observed depending on the geometrical size of the obstacle and on the excitability of the medium: (i) The wave can collide with the obstacle and surround it, recovering its previous shape after surpassing it [Figs. 1(a) and 1(b)]. (ii) The wave can collide with the obstacle and follow its edge while the curvature of the obstacle is small, but suddenly, close to the extreme of the obstacle, the curvature becomes large and the wave will leave the edge, so that a discontinuous wave front is generated, which evolves into a spiral wave [Figs. 1(c) and 1(d)]. Once a spiral is created in a medium, it remains forever, disturbing the propagation of the rest of the waves in that medium.

The different behaviors that we have just related are due to the relationship between the curvature imposed by the obstacle at the extremes and the excitability of the medium. For a particular obstacle, there is a critical velocity of the front close to the obstacle V_c (corresponding to a critical excitability $1/\varepsilon$), below which the wave cannot follow the edge. In the same way, for a given excitability, we observed a critical curvature K_c, above which the wave must leave the edge of the obstacle. In Fig. 2(a), the critical curvature and the critical velocity were plotted as a function of the excitability of the system. In our experimental BZ reaction, we have defined the excitability as $1/\varepsilon=10^2\,[BrO_3^-][H^+]/[CH_2(COOH)_2]$ following [29]. It is possible to observe both V_c and K_c increase with $1/\varepsilon$, but only following a different functional dependence, as can be observed in Fig. 2(b), where V_c is plot-

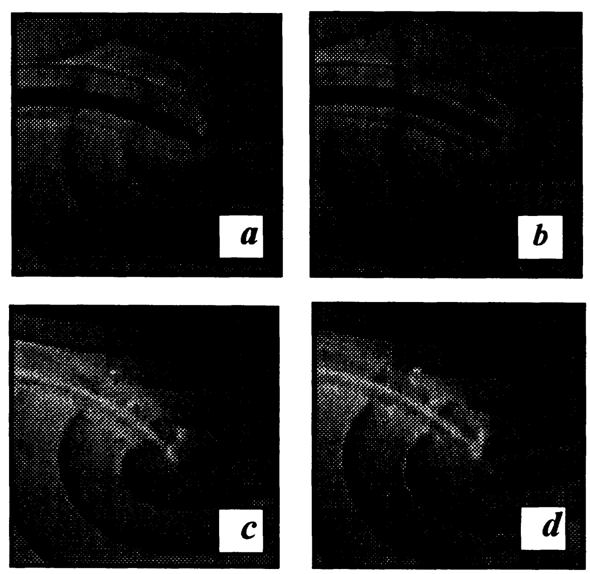

FIG. 1. *Experimental interaction between a wave train and an obstacle.* For small curvatures, the wave that collides with an obstacle (a) is able to surround it (b). The radius of the obstacle is 150 μm and $1/\varepsilon=15$. For large curvatures, a wave that collides with an obstacle (c) cannot follow the edge of the obstacle, giving rise to a discontinuous wave front (d), which will evolve into a rotating spiral wave. The radius of the obstacle is 75 μm and $1/\varepsilon=30$. The obstacle is seen as a stripe at the left of the figures (dark in a and b and lighter in c and d). In a narrow region, close to the obstacle, capillarity effects on the liquid and gel layers can be observed.

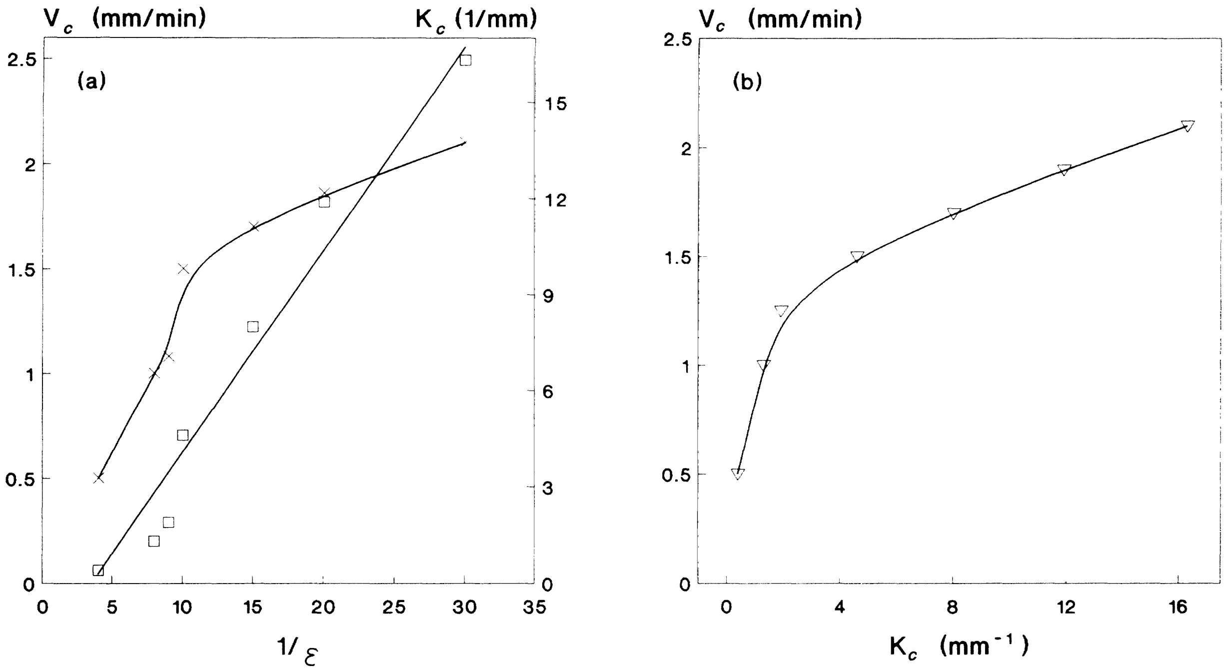

FIG. 2. *Experimental data relative to critical curvature, critical velocity, and excitability.* (a) Both critical velocity (V_c) and curvature (K_c) are shown as a function of the excitability of the medium ($1/\varepsilon$). Note that the functional dependence is different for both variables. While the critical curvature is almost linear in the excitability, the critical velocity seems to saturate for high excitabilities. The critical velocity is represented as X and the curvature as □. (b) V_c is shown as a function of K_c. As it is foreseeable from (a), its dependence is clearly not linear. The excitability of the system is responsible for such behavior.

ted as a function of K_c. The dependence between both variables is clearly not linear, since it is influenced by the excitability of the medium, this influence being stronger for media of low stability.

IV. DISCUSSION

We have investigated the relationship between the critical curvature and the critical velocity for different excitabilities in the framework of the BZ reaction. There exist in the literature different approaches to the same phenomenon. Foerster, Müller, and Hess [23] estimated the critical radius and the diffusion coefficient of a similar BZ medium. Their approach was different because they used a medium with a constant excitability and fitted their results to the well known eikonal equation:

$$V_n = V_0 - DK \ , \tag{1}$$

where V_n is the normal velocity of a wave front with curvature K, V_0 is the normal velocity of a flat front, and D is the diffusion coefficient of the medium. They estimated a value of $D = 1.9 \times 10^{-5}$ cm^2/s. Note that they used a liquid medium, where in a first approach all variables can be considered to have the same diffusion coefficients.

In our experimental approach, we varied two parameters, which led us to the situation described by Zykov's equation [21]. We observed [Fig. 2(b)], in media with different excitabilities, that the dependence between curvature and velocity is not linear. Zykov [21] calculated for a general reaction-diffusion system (the Oregonator model [29] that describes our experiments with the immobilized catalyst can be reduced to that form by scaling time and space) an expression for the critical velocity [Eq. (10) in [21]]

$$V_c \propto (\theta_0 - \theta_1 \varepsilon + DK_c) \ , \tag{2}$$

where θ_0 is the maximal velocity that a flatfront can reach in a medium with $\varepsilon \to 0$ (the system with the highest excitability), and θ_1 is a first-order correction of the velocity for $\varepsilon \neq 0$, which must verify $\theta_1 \leq \theta_0/\varepsilon$. Our experiments fit to Eq. (2) with $D = 2.0 \times 10^{-5}$ cm^2/s, $\theta_0 = 3.8 \times 10^{-3}$ cm/s, and $\theta_1 = 9.9 \times 10^{-4}$ cm/s. The regression coefficients always showed an accuracy better than 97%.

In summary, we have experimentally observed that in a medium with some inhomogeneities (obstacles), spiral formation is intimately related to the shape of those obstacles—the curvature induced on the propagating wave fronts. We have also observed that there exists a relationship between the maximal curvature permitted by different media and the critical velocity in such media. This rate depends on the excitability of every medium, as is predicted by Eq. (2).

The estimated diffusion coefficient agrees quantitatively with that estimated by other authors [23,30], and the maximal velocity of a flatfront in a medium when $\varepsilon \to 0$ (θ_0), has a value higher than the velocity value experimentally observed for the system with the highest excitability.

Note that Eq. (2) was obtained by considering high excitabilities (small values of ε). This equation has been shown to correctly fit our experimental results, but for smaller values of $1/\varepsilon$ its validity must be put under consideration, and it would be necessary to consider a quadratic approach in ε ($\theta_0-\theta_1$, $\varepsilon+\theta_2\varepsilon^2$) to estimate the critical velocity value. Nevertheless, experimental measurements were not carried out for smaller excitabilities, since the system becomes hardly excitable and new problems such as front instabilities make any further study in this region difficult. Our results, showing the influence of the excitability on the critical velocity, can be extent to other systems, e.g., the cardiac muscle, where a similar parameter ε can be defined.

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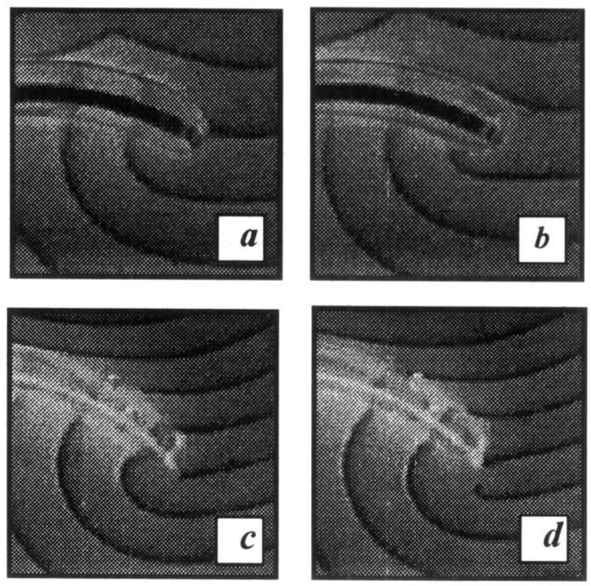

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